NIM : 205314020

Nama : FX Bima Yudha Pratama

Selesaikan relasi rekurensi berikut melalui persamaan karakteristiknya:

1. an = 2an-1 - an-2 untuk n2; dengan kondisi awal a0=1 dan a1=2

2. Relasi Fibonacci fn = fn-1 - fn-2 untuk n2; dengan kondisi awal f0=1 dan f1=1

3. an - 3an-1 + 3an-2 - an-3 = 0 untuk n3; dengan kondisi awal a0=1 dan a1=2 dan a2=4

4. an - 7an-1 + 16an-2 - 12an-3 = 0 untuk n3; dengan kondisi awal a0=1 dan a1=2 dan a2=8

Jawaban

1. an = 2an-1 - an-2 untuk n2; dengan kondisi awal a0=1 dan a1=2

an = 2an-1 - an-2 = an – **2**an-1 + an-2

= r2 – 2r + 1 = 0

=(r – 1) (r – 1)

=r1 = 1 r2 = 1

=a1 = a2 = 1

=an = (c1 + c2n) a1n

=an = (c1 + c2n) 1n

= a0 = 1 ➔ 1 = (c1 + c2(0)) 10

1 = c1

= a0 = 1 ➔ 2 = (c1 + c2(1)) 11

2 = 1 + c2

1 = c2

an = (c1 + c2n) 1n

an = (1 + n) 1n

1. Relasi Fibonacci fn = fn-1 – fn-2 untuk n ≥ 2, f0 = 1, f1 = 2

r2 – r – 1 = 0

Akar-akar karakteristiknya yaitu:

r1 = , r2 =

Solusi relasi rekursif berbentuk : fn = c1r1n + c2r2n =

c1 n + c2n

Dari kondisi awal diperoleh:

c1 + c2 = 1

c1 2 + c2 2 = 1

c1 =

c2 = –

Maka, diperoleh solusi khususnya :

Fn = n – n

1. an – 3an-1 + 3an-2 – an-3 = 0 untuk n ≥ 3, a0 = 1. a1 = 2, a2 = 4

r3 – 3r2 + 3r – 1 = 0

(r – 1) (r – 1) (r – 1)

r1 = 1, r2 = 1, r3 = 1

a1 = a2 = a3 = 1

an = (c1 + c2n + c3n2) a1n

an = (c1 + c2n + c3n2)) 1n

a0 = 1 ➔ 1 = (c1 + c2(0) + c3(0)2) 10

1 = c1

a1 = 2 ➔ 2 = (c1 + c2(1) + c3(1)2) 11

2 = c1 + c2 + c3

2 = 1 + c2 + c3

1 = c2 + c3

a2 = 4 ➔ 4 = (c1 + c2(2) + c3(2)2) 12

4 = (1 + 2c2 + 4c3)

3 = 2c2 + 4c3

c2 + c3 = 1

2c2 + 4c3 = 3

c1 = 1 c2 = ½ c3 = ½

an = (c1 + c2n + c3n2)) 1n

an = (1 + (½)n + (½)n2) 1n

1. an = 7an-1 + 16an-2 – 12an-3 = 0 untuk n ≥ 3, a0 = 1, a1 = 2, a2 = 2, a3 = 8

r3 – 7r2 + 16r – 12 = 0

(r – 2) (r – 2) (r – 3)

a1 = a2 = 2

a3 = 3

an = (c1 + c2n + c3n2) a1n

an = (c1 + c2n + c3n2) 1n

a0 = 1 ➔ 1 = (c1 + c2(0) +c3(0)2) 10

1 = c1

a1 = 2 ➔ 2 = (1 + c2(1) + c3(1)2) 11

2 = (1 + c2 + c3)

1 = c2 + c3

a2 = 8 ➔ 8 = (1 + c2(2) + c3(2)2) 12

8 = (1 + 2c2 + 4c3)

7 = 2c2 + 4c3

c2 + c3 = 1

2c2 + 4c3 = 7

c1 = 1

c2 = 3/2

c3 = -5/2

an = (c1 + c2n + c3n2) 1n

an = (1 + 3/2n + -5/2n2) 1n